Name solutions

EE 311 Final Exam Fall 2010

December 16, 2010

Closed Text and Notes, No calculators

1) Be sure you have 13 pages and the additional pages of equations.

2) Write only on the question sheets. Show all your work. If you need more room for a particular problem, use the reverse side of the same page.

3) Write neatly, if your writing is illegible then print.

5) This exam is worth 150 points.

(5 pts) 1. A point charge of 30 nC is located at (-1, 0, 2), -20 nC at (0, 0, 0) and 10 nC at (1, 5, -1) The total electric flux leaving a cube of side 6 centered at the origin is

A) – 20 nC

[✓ B)] 10 nC

C) 20 nC

D) 30 nC

E) 60 nC

(20 pts) 2. Circle true or false concerning the statements for a ferromagnetic material.

The electric flux density on a spherical surface r = b is the same for a point charge Q located at the origin and for charge Q uniformly distributed on surface r = a where a < b.	[✓ True]	False
Energy cannot be stored in an inductor.	True	[✓ False]
The inductance of a coil is independent of the current flowing through the coil.	[✓ True]	False
A ferromagnetic material retains a considerable amount of its magnetization when removed from an externally applied field.	[✓ True]	False
Electric flux lines are never closed.	True	[✓ False]
Two thin parallel wires carry currents along the same direction. The force experienced by one due to the other is perpendicular to the wires and attractive.	[✓ True]	False
Displacement current is caused by a changing electric field.	[✓ True]	False
The integration of the Poynting vector over any closed surface gives the net power flowing out of that surface.	[✓ True]	False
A standing wave occurs when the electric and magnetic fields of a TEM wave are out-of-phase.	True	[✓ False]
The critical angle is where the angle of incidence equals the angle of reflectance.	True	[✓ False]

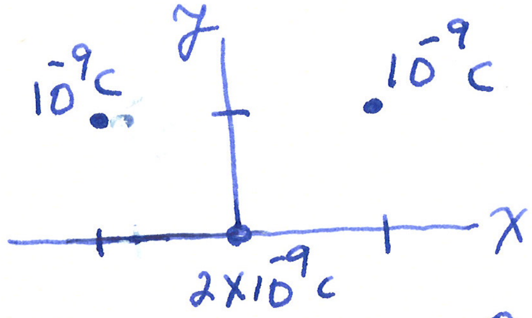
(5 pts) 3. A 5 μC point charge is located at (1, π/4, π/2) and experiences a force of $\mathbf{F} = 10\hat{\mathbf{a}}_r - 5\hat{\mathbf{a}}_\theta + 25\hat{\mathbf{a}}_\phi$ N. What is the electric field intensity at point (1, π/4, π/2)?

$$\vec{E} = \frac{\vec{F}}{Q} = \frac{(10\hat{a}_r - 5\hat{a}_\theta + 25\hat{a}_\phi)N}{5\times10^{-6}\,C}$$

$$= (2\hat{a}_r - \hat{a}_\theta + 5\hat{a}_\phi)\times10^{6}\,\frac{N}{C}$$

$$\vec{E} = (2\hat{a}_r - \hat{a}_\theta + 5\hat{a}_\phi)\times10^{6}\,\frac{V}{m}$$

(5 pts) 4. There is a 1 nC point charge at (-1, 1, 0), a 2 nC charge at (0, 0, 0), and a 1 nC point charge at (1, 1, 0). Choose $V(r=\infty) = 0$ and use ε_o in the form $\varepsilon_o = \frac{10^{-9}}{36\pi}\,\frac{F}{m}$ in your calculations. V(0, 1, 0) is



A) 0V

B) ∞ V

C) 4 V

D) 1 V

[✓ E) 36 V]

F) 18 V

G) 9 V

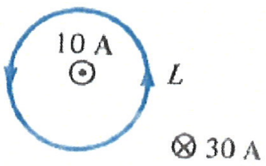
H) none of the above

$$V(0,1,0) = \frac{1}{4\pi\varepsilon_o}\sum_i \frac{Q_i}{r_i}$$

$$= \frac{1}{4\pi\,\frac{10^{-9}}{36\pi}\,\frac{F}{m}}\left(\frac{10^{-9}c}{1m} + \frac{2\times10^{-9}c}{1m} + \frac{10^{-9}c}{1m}\right)$$

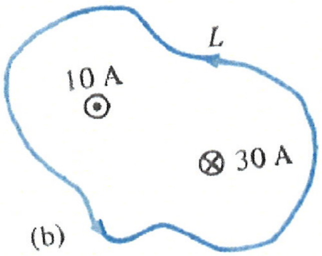
$$= \left(9\,\frac{m}{F}\right)\left(4\,\frac{c}{m}\right) = 36V$$

(8 pts) 5. For the currents and closed paths shown in the following four figures, evaluate $\oint \mathbf{H} \bullet \mathbf{dl}$



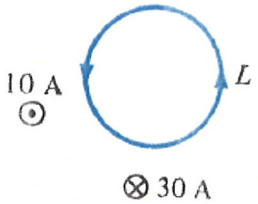
(a)

$\oint \mathbf{H} \bullet \mathbf{dl} = 10A$



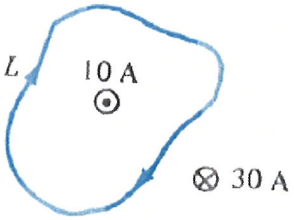
(b)

$\oint \mathbf{H} \bullet \mathbf{dl} = -20A$



(c)

$\oint \mathbf{H} \bullet \mathbf{dl} = 0$



(d)

$\oint \mathbf{H} \bullet \mathbf{dl} = -10A$

(5 pts) 6. Which of the following statements is correct?

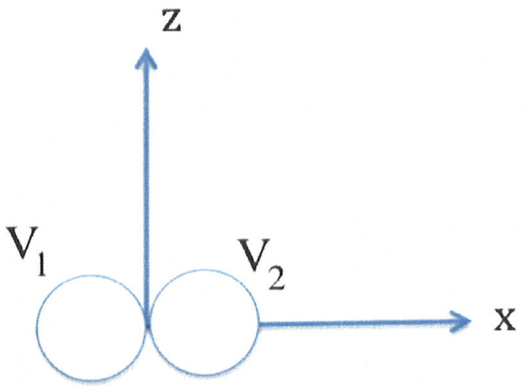
[✓ a)] The magnetic flux density is due to free and bound currents, the magnetic field intensity is due to free currents, and the magnetization is due to bound currents.

b) The magnetic flux density is due to free currents, the magnetic field intensity is due to free and bound currents, and the magnetization is due to bound currents.

c) The magnetic flux density is due to bound currents, the magnetic field intensity is due to free currents, and the magnetization is due to free and bound currents.

d) The magnetic flux density is due to free currents, the magnetic field intensity is due to bound currents, and the magnetization is due to free and bound currents.

(10 pts) 7. Two identical spherical conductors of radius a = 1m are brought into contact at a single point P. A total charge of Q = -1 nC is placed on the conductors such that the potentials for the two conductors become V_1 and V_2 .



(3 Pts) a) What is the relationship between V_1 and V_2 ?

conductors are equipotentials. Since these two conductors are in contact, they are at the same potential $V_1 = V_2$

(2 Pts) b) Find the electric field inside the two contacting spheres.

There is no charge inside these conductors, so there is no electric field inside $E = 0$

(5 pts) c) Find the total electric field at (s, y, z) = (o, 0, 10 km). Hint, take advantage of 10 km >>> 1 m and use ε_0 in the form $\varepsilon_0 = \frac{10^{-9}}{36\pi}\frac{F}{m}$ in your calculations.

From 10 km away, these spheres will look like a point charge of -1nC at the origin.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{a}_r = \frac{1}{4\pi \frac{10^{-9}}{36\pi}\frac{F}{m}} \frac{-10^{-9}c}{(10^4 m)^2}\hat{a}_r$$

$$= -9\times10^{-8}\,\hat{a}_r \frac{V}{m}$$

(15 pts) 8. A parallel plate capacitor has plate area $0.5\ \text{m}^2$, plate separation $0.02\ \text{m}$, and a dielectric between the plates of relative permittivity $\varepsilon_r = 4$. A battery is connected to charge the positive plate with $8.854\text{x}10^{-9}\,\text{C}$ and the negative plate with $-8.854\text{x}10^{-9}\,\text{C}$. The plates are parallel to the yz plane and the positive plate is at x = 0 and the negative plate at x = 0.02 m. Ignore field fringing at the edges of the capacitor and $\varepsilon_o = 8.854\text{x}10^{-12}\,\frac{\text{F}}{\text{m}}$.

(5 Pts) a) What is the electric flux density inside the capacitor?

$$D = \rho_s = \frac{Q}{A} = \frac{8.854 \times 10^{-9} C}{0.5\, m^2} = 17.708 \times 10^{-9} \frac{C}{m^2}$$

$$\vec{D} = 1.7708 \times 10^{-10} \frac{C}{m^2} \hat{a}_x$$

(5 Pts) b) What is the electric field intensity inside the capacitor?

$$\vec{E} = \frac{\vec{D}}{\varepsilon_r \varepsilon_o} = \frac{17.708 \times 10^{-9} \frac{C}{m^2} \hat{a}_x}{(4)(8.854 \times 10^{-12} \frac{F}{m})} = 10^3 \hat{a}_x \frac{C}{mF}$$

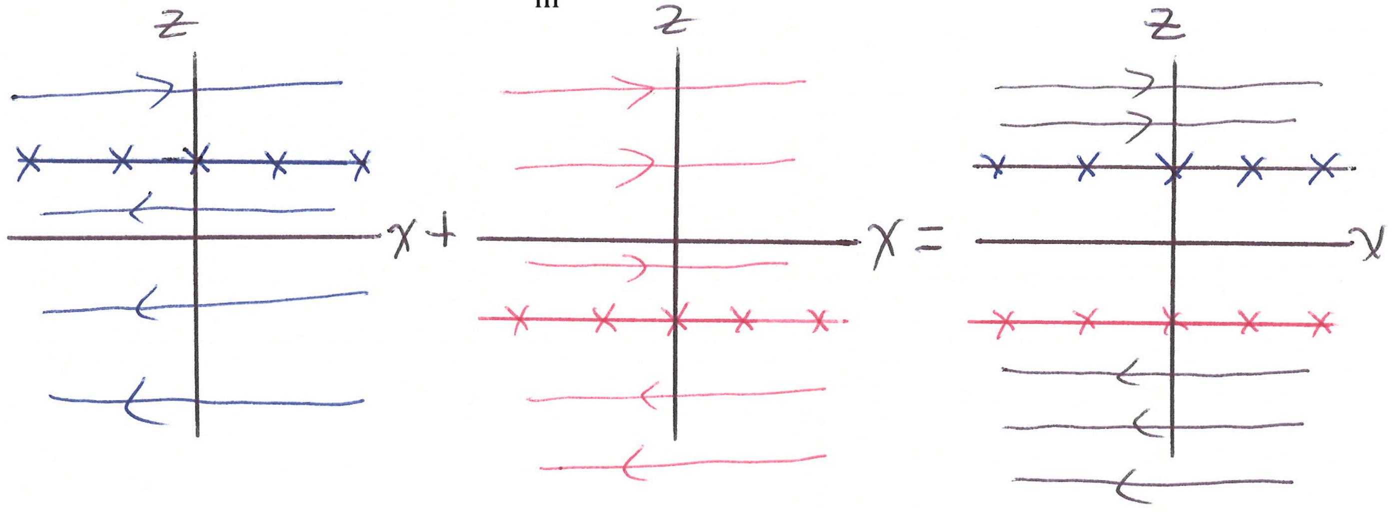
$$\vec{E} = 500 \hat{a}_x \frac{V}{m}$$

(5 Pts) c) What is the voltage drop across the capacitor

$$V = Ed = \left(500 \frac{V}{m}\right)(0,02 m)$$

$$= 10\ V$$

(12 pts) 9 The z = 1 plane contains a sheet current density of $\mathbf{K} = 10\,\frac{\text{A}}{\text{m}}\,\hat{\mathbf{a}}_y$. The z = -1 plane contains a sheet current density of $\mathbf{K} = 10\,\frac{\text{A}}{\text{m}}\,\hat{\mathbf{a}}_y$. Find the magnetic field intensity everywhere.



$$\vec{H} = 5\hat{a}_x \frac{A}{m},\ z>1$$
$$= -5\hat{a}_x \frac{A}{m},\ z<1$$

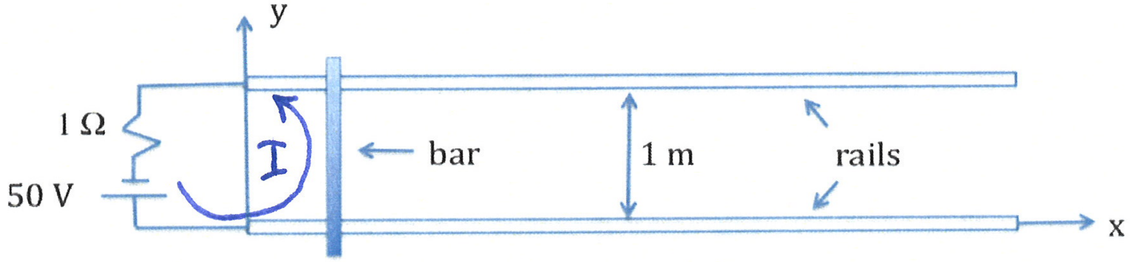
+

$$\vec{H} = 5\hat{a}_x \frac{A}{m},\ z>-1$$
$$= -5\hat{a}_x \frac{A}{m},\ z<-1$$

=

$$\vec{H} = 10\hat{a}_x \frac{A}{m},\ z>1$$
$$= 0,\ -1<z<1$$
$$= -10\hat{a}_x \frac{A}{m},\ z<-1$$

(10 pts) 10. A 1 kg sliding bar is on the rail system shown. The magnetic flux density is everywhere $\mathbf{B}=1\hat{\mathbf{a}}_z\,\mathrm{T}$, If the bar starts at rest at x = 0, what is the force on the sliding bar? Assume the resistances of the rails and the sliding bar are negligible.



$$I = \frac{50\,V}{1\,\Omega} = 50\,A$$

$$\vec{F} = I \vec{L} \times \vec{B}$$

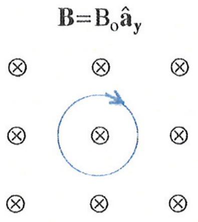
$$= (50A)(1m\,\hat{a}_y) \times \left(1\,\hat{a}_z \frac{Wb}{m^2}\right)$$

$$= 50\,\hat{a}_x \frac{A\,Wb}{m}$$

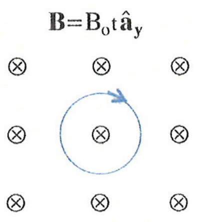
$$\vec{F} = 50\,\hat{a}_x\,N$$

$$\left(\frac{A\,Wb}{m} = \frac{A\,V\,s}{m} = \frac{C\,J\,s}{s\,C\,m} = \frac{J}{m} = \frac{Nm}{m} = N\right)$$

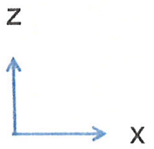
(12 pts) 11. Consider the following conducting loops placed in the indicated magnetic fields. If current is flowing in the direction of the arrow, clockwise, the current is considered positive. Indicate whether the actual current flowing is positive, negative, or zero for the indicated conditions. Note the positive y-axis is into the page.

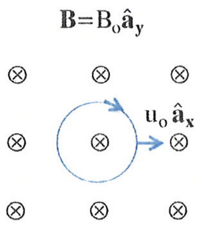


I is zero

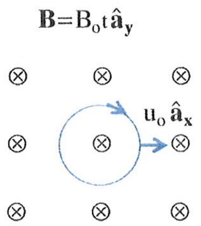


I is negative





I is zero



I is negative

(19 pts) 12. The following is the equation of the electric field intensity of an TEM wave.

$$\mathbf{E} = 35e^{-5z}\sin(4\pi x 10^6 t - 2\pi z)\hat{\mathbf{a}}_x \frac{V}{m}$$

Be sure to include units in the following

(3 pts) What is attenuation constant of the wave?

$\alpha = 5\ m^{-1}$

(4 pts) What is the skin depth?

$\alpha\delta = 1 \qquad \delta = \frac{1}{5 m^{-1}} = 0.2 m$

(4 pts) What is the wave number?

$\beta = 2\pi\ m^{-1}$

(4 pts) What is the wavelength?

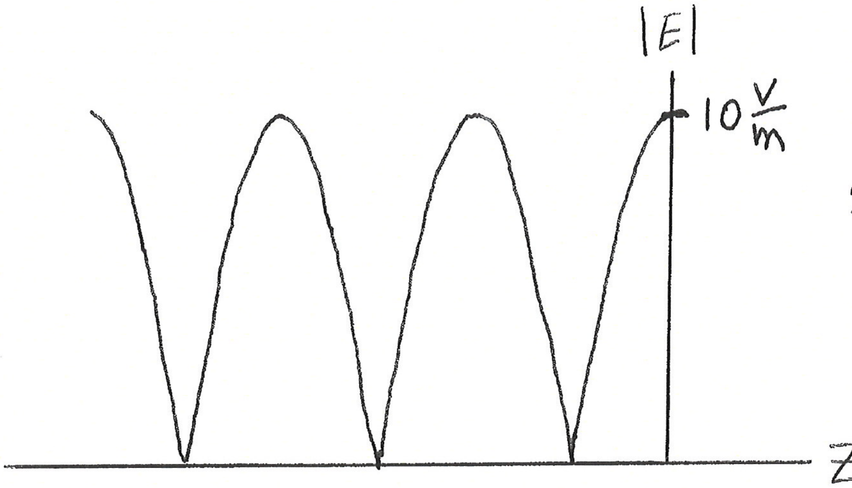
$\beta = \frac{2\pi}{\lambda} = 2\pi \quad \Rightarrow \quad \lambda = 1 m$

(4 pts) What is the velocity of the wave?

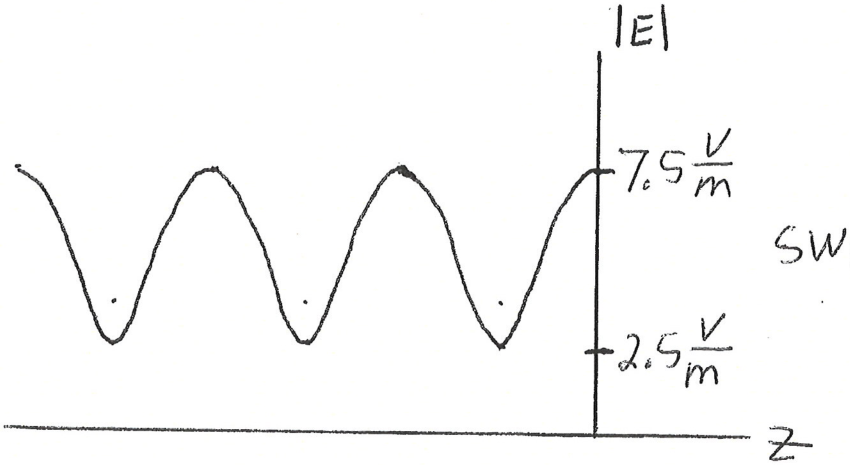
$u = \frac{\omega}{\beta} = \frac{4\pi \times 10^6 s^{-1}}{2\pi\ m^{-1}} = 2 \times 10^6 \frac{m}{s}$

$\vec{u} = 2 \times 10^6 \frac{m}{s}\ \hat{a}_z$

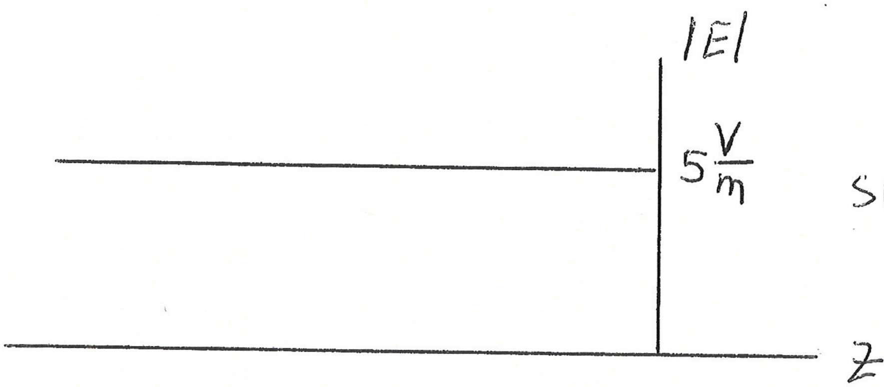
(9 pts) 13. Shown are the amplitudes versus position for three TEM waves. What are the standing wave ratios for each?



SWR= $\frac{10}{0} = \infty$



SWR= $\frac{7.5}{2.5} = 3$



SWR= 1

(15 pts) 14. A TEM uniform plane wave is propagating in the +z-direction. The amplitude of the electric field is $\mathbf{E}=10\hat{\mathbf{a}}_y \frac{\mathrm{V}}{\mathrm{m}}$, the wave velocity is $3\mathrm{x}10^8 \frac{\mathrm{m}}{\mathrm{s}}$, and the radian frequency is $\omega=3\sqrt{2}\mathrm{x}10^{10}\,\mathrm{s}^{-1}$.

(5 pts) a)Write the equation describing the electric field. (Not the phasor representation but the actual representation of the electric field.)

$$\beta = \frac{\omega}{u} = \frac{3\sqrt{2}\times 10^{10}\, s^{-1}}{3\times 10^{8}\, m/s} = \sqrt{2}\times 10^{2}\, m^{-1}$$

$$\vec{E}(z,t) = 10\cos\left(3\sqrt{2}\times 10^{10} t - \sqrt{2}\times 10^{2} z\right) \hat{a}_y \frac{V}{m}$$

(10 pts) b) Write the equation describing the electric field if the amplitude of the electric field is still $\mathbf{E}=10\hat{\mathbf{a}}_y \frac{\mathrm{V}}{\mathrm{m}}$, but the direction of propagation is in the direction $\frac{1}{\sqrt{2}}\hat{\mathbf{a}}_x + \frac{1}{\sqrt{2}}\hat{\mathbf{a}}_z$.

$$\vec{k} = k_x \hat{a}_x + k_y \hat{a}_y = \beta \hat{a}_k = (\sqrt{2}\times 10^{2})\left(\frac{1}{\sqrt{2}}\hat{a}_x + \frac{1}{\sqrt{2}}\hat{a}_y\right)$$

$$= 100\hat{a}_x + 100\hat{a}_y$$

$$\vec{E}(x,z,t) = 10\cos\left(3\sqrt{2}\times 10^{10} t - 100x - 100z\right)\hat{a}_y \frac{V}{m}$$